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EVOLUTION OF THREE-DIMENSIONAL GRAVITATIONALLY WARPED WAVES DURING THE MOVEMENT OF A PRESSURE ZONE OF VARIABLE INTENSITY

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Three-dimensional, unestablished, gravitationally warped waves arising due to the motion of a harmonically time-varying pressure zone over a solid, thin plate floating on the surface of a homogeneous liquid of finite depth have been studied in the linear formulation. In the absence of a plate, three-dimensional waves are generated by the movement of a region of periodic perturbations, where established waves have been studied in [1, 2], and unestablished waves have been investigated in [3-5]. The evolution of three-dimensional, gravitationally warped waves formed during the motion of a constant load over a plate has been considered in [6].

1. We will consider a homogeneous, ideal, incompressible liquid of finite depth H covered by a thin, elastic plate. Beginning from time t = 0, a force of the following form acts on the surface of the plate:

$$p = p_0 f(x_1, y)\exp(i\sigma t),\ x_1 = x + vt,\ v = \text{const}. \tag{1.1}$$

We will investigate the evolution of excited wave motion assuming that the liquid is unperturbed up until the time when the force (1.1) acts and that the interface between the plate and the liquid (the flexure of the plate) ζ is horizontal.

Considering the motion of the liquid to be that of a potential and the velocity of the particles of the liquid and the elevation of the liquid–plate interface to be small, we will find in the coordinate system x_1, y, which is connected to a pressure zone moving with a velocity v, the velocity potential φ through the Laplace equation

$$\Delta\varphi = 0,\ -H < z < 0,\ -\infty < x < \infty,\ -\infty < y < \infty \tag{1.2}$$

with the following boundary and initial conditions

$$D_1\nabla^4\zeta + \varkappa_1 F\zeta + \zeta + (\varphi_t + v\varphi_x)\frac{1}{g} = -\frac{p}{\rho g}\quad (z = 0), \tag{1.3}$$
$$\varphi_z = 0\ (z = -H),\ \varphi(x, y, z, 0) = \zeta(x, y, 0) = 0,$$
$$D_1 = \frac{D}{\rho g},\quad \varkappa_1 = \frac{\rho_1 h}{\rho g},\quad D = \frac{Eh^3}{12(1-\mu^2)},\quad \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4},$$
$$F = \frac{\partial^2}{\partial t^2} + 2v\frac{\partial^2}{\partial t\partial x} + v^2\frac{\partial^2}{\partial x^2},$$

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where ρ is the density of the liquid; E, H, ρ_1, and μ are the modulus for normal elasticity, the thickness, the density, and the Poisson coefficient for the plate; ζ and φ for $z = 0$ are related by the condition $\zeta_t = \varphi_z - v\zeta_x$. We will from now on omit the subscript 1 from the variable x_1.

Using Fourier transformations with respect to x, y and the Laplace transformation with respect to t from (1.2), (1.3) for the elevation of the plate–liquid interface, we find that

$$\zeta = \frac{a}{8\pi^2} e^{i\sigma t} \int\limits_{-\infty}^{\infty}\int f^*(m, n) M(r) \psi(m, n, t) e^{i(mx+ny)} dm dn,$$

$$\psi = \frac{2}{\Delta_1\Delta_2} - \frac{1}{\tau\Delta_1} e^{-i\Delta_1 t} + \frac{1}{\tau\Delta_2} e^{-i\Delta_2 t}, \quad a = \frac{p_0}{\rho g},$$

$$\tau(r) = [l(r)M(r)]^{1/2}, \; l(r) = 1 + D_1 r^4, \; r = (m^2 + n^2)^{1/2},$$

$$M(r) = rg(1 + \varkappa_1 rg \operatorname{th} rH)^{-1} \operatorname{th} rH, \; \Delta_j = \sigma + vm + \delta_j\tau, \; \delta_j = (-1)^j,$$

$f^*(m, n)$ is the transformant of the Fourier function $f(x, y)$. The first term in the expression for ζ is the solution of the problem without the initial conditions, i.e., it is representative of established oscillations. The second and third terms are determined by the initial conditions and characterize the evolution of wave motion.

Since

$$\psi = \frac{1}{\tau}\left[\frac{1}{\Delta_1}\left(1 - e^{-i\Delta_1 t}\right) - \frac{1}{\Delta_2}\left(1 - e^{-i\Delta_2 t}\right)\right], \quad \int\limits_0^t e^{-i\Delta_j\xi} d\xi = -i\left(1 - e^{-i\Delta_j t}\right)/\Delta_j,$$

then ζ has the following form after making a transition to polar coordinates for an axially symmetric pressure distribution (1.1):

$$\zeta = -\frac{a}{8\pi^2} \operatorname{Im}\left[e^{i\sigma t}(J_1 - J_2)\right], \tag{1.4}$$

$$J_j = \int\limits_0^\infty \int\limits_0^t \int\limits_{-\pi/2}^{3\pi/2} \frac{rf^*(r)}{\tau(r)} M(r) e^{-i[rR\cos(\theta-\gamma) - \Delta_j\xi]} d\theta d\xi dr,$$

$$R = (x^2 + y^2)^{1/2}, \; x = R\cos\gamma, \; y = R\sin\gamma, \; m = r\cos\theta, \; n = r\sin\theta.$$

One can investigate the asymptotic behavior of the expression for ζ for large values of R and t by using the steady-state phase method for multidimensional integrals. Steady-state points (r, θ, ξ) for J_j satisfy the following system of equations

$$R\cos(\theta - \gamma) - (v\cos\theta + \delta_j\tau')\xi = 0, \; v\xi\sin\theta - R\sin(\theta - \gamma) = 0; \tag{1.5}$$

$$\sigma + \delta_j\tau + vr\cos\theta = 0, \tag{1.6}$$

where the primes denote differentiation with respect to r.

Equation (1.6) has the following real roots:

$$\theta = \pm\theta_j, \; \theta_j = \arccos(-\delta_j\tau_j), \; \tau_j = (\tau + \delta_j\sigma)/(vr)$$

for $|\tau_j| \leq 1$. After substituting $\theta = \pm\theta_j$ into (1.5), we find that

$$\xi = R/u_j, \; u_j = (v - \delta_j\tau')\cos^{-1}\gamma, \tag{1.7}$$

$$\operatorname{tg}\gamma = \chi_j(r), \quad \chi_j = \tau'\left(1 - \tau_j^2\right)^{1/2}(v - \tau_j\tau')^{-1}; \tag{1.8}$$

Equation (1.7) can be used to determine the values of r that correspond to the steady-state points of the integrals J_j in the established and unestablished modes. The applicability of the steady-state points of the integration region is determined by the conditions $0 \leq \xi \leq t$. This condition along with relation (1.8), characterizes the propagation of the oscillations.

TABLE 1

v	γ	k	σ
$0<v<v_{01}$	$0\leqslant\gamma\leqslant\pi$	4	
$v_{01}<v<v_{11}$	$0\leqslant\gamma\leqslant\pi$	3, 4	$0<\sigma<\sigma_0$
$v_{11}<v<v_{02}$	$0\leqslant\gamma<\gamma_2$	1, 4	
	$\gamma_2<\gamma<\gamma_1$	1, 2, 3, 4	
	$\gamma_1<\gamma\leqslant\pi$	3, 4	
$v_{02}<v$	$0\leqslant\gamma<\gamma_2$	4	
	$\gamma_2<\gamma<\gamma_3$	2, 3, 4	
	$\gamma_3<\gamma\leqslant$	3	
$0<v<v_{11}$	$0\leqslant\gamma\leqslant\pi$	4	
$v>v_{11}$	$0\leqslant\gamma<\gamma_2$	4	
	$\gamma_2<\gamma<\gamma_3$	2, 3, 4	$\sigma>\sigma_0$
	$\gamma_3<\gamma\leqslant\pi$	3	

TABLE 2

v	γ	k	σ
$v_{03}<v<v_{12}$	$0\leqslant\gamma\leqslant\pi$	3	
$v>v_{12}$	$0\leqslant\gamma<\gamma_{22}$	1	$\sigma>0$
	$\gamma_{22}<\gamma<\gamma_{11}$	1, 2, 3	
	$\gamma_{11}<\gamma\leqslant\pi$	3	

The behavior of the function $\chi_1(r)$ for $0 < \sigma < \sigma_0$ and of the function $\chi_2(r)$ for $\sigma > 0$ is qualitatively shown in Fig. 1, where a-g corresponds to $0 < v < v_{01}$, $v_{01} < v < v_{11}$, $v_{11} < v < v_{02}$, $v_{02} < v < v_{10}$, $v > v_{10}$, $v_{03} < v < v_{12}$, $v > v_{12}$, respectively. If $\sigma > \sigma_0$, the plots of $\chi_1(r)$ for $0 < v < v_{11}$ and $v > v_{11}$ are similar to the plots of $\chi_2(r)$ for $v_{03} < v < v_{02}$ and $v > v_{12}$. Here,

$$\sigma_0 = [\beta_2\tau(\beta_1) - \beta_1\tau(\beta_2)](\beta_2 - \beta_1)^{-1},\ v_{01} = \tau_5(\beta_2),$$
$$v_{02} = \tau_5(\beta_1),\ v_{03} = \tau_6(\beta_3),\ v_{11} = [\tau_7(\beta_4)]^{1/2},\ v_{12} = [\tau_8(\beta_6)]^{1/2},$$
$$v_{10} = [\tau_9(\beta_8)]^{1/2},\quad \tau_{5,6} = (\tau \pm \sigma)/r,\quad \tau_7 = \tau_5^2 - r(\tau_5')^2\tau'/\tau'',$$
$$\tau_8 = \tau_6^2 - r(\tau_6')^2\tau'/\tau'',\quad \tau_9 = \tau_5\tau',\quad \tau_5'(\beta_{1,2}) = 0,$$
$$\tau_6'(\beta_3) = 0,\quad \chi_1''(\beta_{4,5}) = 0,\quad \chi_2''(\beta_{6,7}) = 0,\quad \tau_9'(\beta_{8,9}) = 0,$$
$$\beta_1 < \beta_2,\ \beta_4 < \beta_5,\ \beta_6 < \beta_7,\ \beta_8 < \beta_9,$$
$$r_1 < r_2 < \beta_1 < r_3 < \beta_2 < r_4,\ r_5 < \beta_3 < r_6,$$

where r_k ($k = 1, 2, 3, 4$) are the positive roots of the equation $1 - \tau_1^2 = 0$, and $r_{5,6}$ are the positive roots of the equation $1 - \tau_2^2 = 0$.

It follows from an analysis of the behavior of the functions $\chi_j(r)$ that the number k of positive roots α_{jk} of Eq. (1.7) is a function of the angle γ, the frequency of the oscillations σ, and the displacement velocity v of the pressure. This is evident from Tables 1 and 2, where the values of k for α_{1k} and α_{2k} are indicated. Because of the symmetry of the wave motion relative to the x axis, values of γ are presented in the tables only over the range $0 \leq \gamma \leq \pi$. Here, $\gamma_2 = \arctan\chi_1(\alpha_2)$, $\gamma_{11} = \arctan\chi_2(\alpha_3)$, $\gamma_{22} = \arctan\chi_2(\alpha_4)$, and the equation $\gamma = \arctan\chi_1(\alpha_1)$ is used for determining the angle γ_1 if $v_{11} < v < v_{02}$ and the angle γ_3 if $v > v_{02}$. One should note that

$$\chi_1'(\alpha_{1,2}) = 0,\quad \chi_2'(\alpha_{3,4}) = 0,\quad \alpha_1 < \alpha_2,\quad \alpha_3 < \alpha_4.$$

If $\sigma > k$, the following estimates are valid for α_{2k}:

$$r_5 \leqslant \alpha_{23} \leqslant r_6\ (v_{03} < v < v_{12}),\ r_5 \leqslant \alpha_{21} \leqslant \alpha_3 \leqslant \alpha_{22} \leqslant \alpha_4 \leqslant \alpha_{23} \leqslant$$
$$\leqslant r_6(v > v_{12}).$$

In addition,

$$r_1 \leqslant \alpha_{14} \leqslant r_2(0 < v < v_{01}),\ r_1 \leqslant \alpha_{14} \leqslant r_2 < r_3 \leqslant \alpha_{13} \leqslant r_4(v_{01} < v < v_{11}),$$

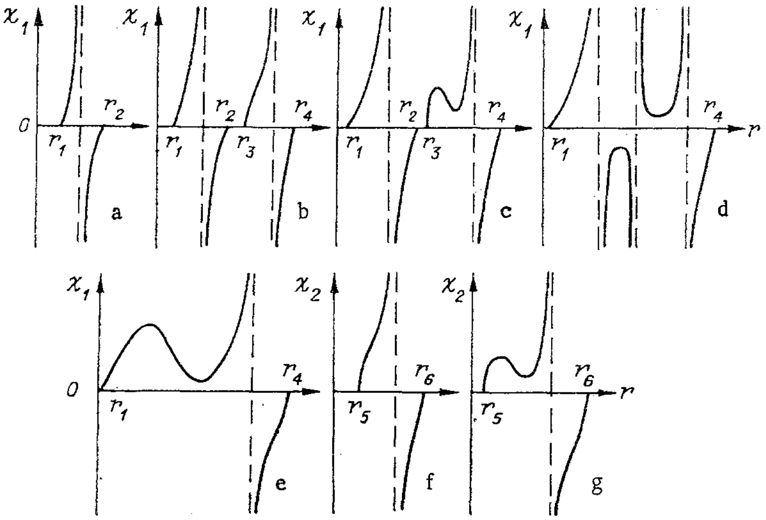

Fig. 1

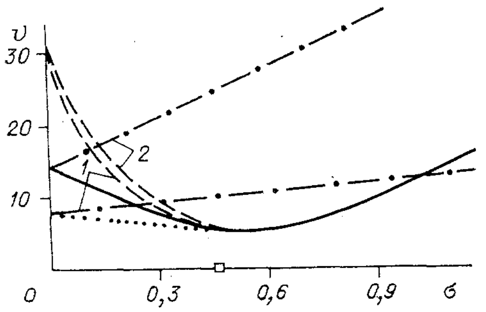

Fig. 2

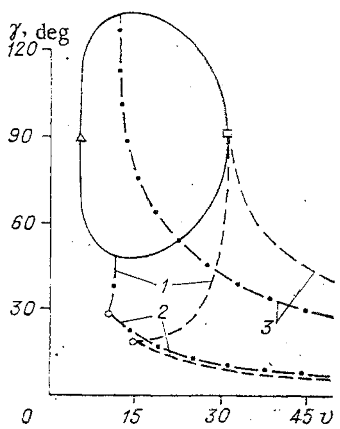

Fig. 3

$$r_1 \leqslant \alpha_{15} \leqslant r_2 < r_3 \leqslant \alpha_{11} \leqslant \alpha_1 \leqslant \alpha_{12} \leqslant \alpha_2 \leqslant \alpha_{13} \leqslant r_4\ (v_{11} < v < v_{02}),$$
$$r_1 \leqslant \alpha_{14} \leqslant \alpha_1 \leqslant \alpha_{12} \leqslant \alpha_2 \leqslant \alpha_{13} \leqslant r_4\ \ (v > v_{02})$$

for $\sigma < \sigma_0$ and

$$r_1 \leqslant \alpha_{14} \leqslant r_2 (0 < v < v_1),\ r_1 \leqslant \alpha_{14} \leqslant \alpha_1 \leqslant \alpha_{12} \leqslant \alpha_2 \leqslant \alpha_{13} \leqslant r_2$$
$$(v > v_{11})$$

for $\sigma > \sigma_0$.

Each root α_{jk} of Eqs. (1.7) characterizes a system of waves ζ_{jk} of the form

$$\zeta_{jk} = \frac{1}{\sqrt{R}} \psi_j(\alpha_{jk}) \cos\left[R\Phi_j(\alpha_{jk}) + \sigma t + \delta_{jk}\frac{\pi}{4}\right] + O\left(\frac{1}{R}\right),$$
$$\psi_j = -af^*(r) M(r) \left[(1-\tau_j^2)\tau v\right]^{-1} (2\pi |\Phi_j''|)^{-1/2},$$
$$\Phi_j = \delta_j r \left[(1-\tau_j^2)^{1/2} \sin\gamma - \tau_j \cos\gamma\right],$$
$$j = 1,\ k = 1-4;\ j = 2,\ k = 1, 2, 3;\ \delta_{jk} = (-1)^{j+k} (k \neq 4),\ \delta_{14} = 1,$$

that are generated in the regions $R < u_{jk}t$, $u_{jk} = u_j(\alpha_{jk})$ of the angular zones corresponding to the variation range of the displacement velocity of the pressure zones (see Tables 1 and 2). The dimensions of the angular zones are given by the values of the angles γ_1, γ_2, γ_3, γ_{11}, γ_{22}.

Therefore, for motion of a pressure zone (1.1), one (ζ_{14}), two (ζ_{13}, ζ_{14}; ζ_{14}, ζ_{23}), three (ζ_{23}, ζ_{1k}, k = 3, 4; ζ_{1k}, k = 2-4), four (ζ_{1k}, k = 1-4; ζ_{23}, ζ_{1k}, k = 2-4), five

(ζ_{23}, ζ_{1k}, k = 1-4), six (ζ_{1k}, k = 2-4, ζ_{2k}, k = 1-3), or seven (ζ_{1k}, k = 1-4, ζ_{2k}, k = 1-3) systems of waves can be excited with amplitudes on the order of $R^{-1/2}$. They generate the oscillation of the plate and the wave motion of the liquid before and after the pressure zone.

The waves ζ_{14}, ζ_{21}, ζ_{22} are caused by the periodic changes in the pressure over time ($\sigma > 0$). Then, ζ_{21} behave as transverse waves and ζ_{22} behave as longitudinal ship waves generated in the corresponding angular zones beyond the moving zone of periodic pressure. However, for an elastic plate ($E > 0$), these waves are formed when $v > v_{12}$, and for a liquid with an absolutely fragile plate ($E = 0$), or with an open surface ($h = 0$), they are formed when $v > 0$.

The waves ζ_{14} are generated when $v > 0$, and when the zone of periodic pressure is not displaced [7]. They are circular in form. These waves are also formed in a liquid with an open surface or with an absolutely fragile plate. Depending on the velocity v, the waves ζ_{14} may be found around the pressure zone ($v < v_{02}$) or in the angular zone $|\gamma| < \gamma_3$ beyond the zone ($v > v_{02}$). If $v_{02} < v < v_{10}$, the waves ζ_{14} do not form directly before the pressure zone along its direction of motion, but for perturbations caused by the waves ζ_{14} that are parallel to this direction, the waves overtake the pressure zone ($\gamma_3 > \pi/2$) When $v > v_{10}$, this does not occur ($\gamma_3 < \pi/2$).

The waves ζ_{11} and ζ_{12} are transverse and longitudinal waves, respectively, that arise beyond the moving perturbation zone. They also form when the pressure zone moves along the plate (a free surface) with constant intensity [6, 8]. For an elastic plate, ζ_{11} and ζ_{12} are generated for $v_{11} < v < v_{02}$ and $v > v_{11}$. For an absolutely fragile plate or for a free surface, ζ_{11} are formed when $0 < v < v_{02}$, and ζ_{12} are formed when $v > 0$.

The waves ζ_{13} and ζ_{23} are warped. They are generated by the moving pressure zone only in the presence of a solid, elastic plate ($E > 0$). The waves ζ_{13} are formed when $v > v_{01}$ and ζ_{23} are formed when $v > v_{03}$. Of these, ζ_{13} are excited by pressures of variable ($\sigma > 0$) and constant ($\sigma = 0$) intensity, and ζ_{23} are excited only for displacements of pressure that are periodic over time. The direction of waves ζ_{13} and ζ_{23} characterize the angle

$$\gamma_{01} = \operatorname{arc\,tg}\left(\frac{v^2}{v_{01}^2} - 1\right)^{1/2}, \quad \gamma_{02} = \operatorname{arc\,tg}\left(\frac{v^2}{v_{03}^2} - 1\right)^{1/2}.$$

The waves ζ_{13} for $v_{01} < v < v_{11}$ and ζ_{23} for $v_{03} < v < v_{12}$ are found around the pressure zone, while for $v > v_{11}$ and $v > v_{12}$, they are located in the angular zones $\gamma_2 < |\gamma| \leq \pi$ and $\gamma_{22} < |\gamma| \leq \pi$.

The leading perturbation fronts ζ_{jk} (j = 1, k = 1-4/ j = 2, k = 1, 2, 3) are displaced with a velocity of u_{jk}.

2. We considered an ice plate for making quantitative estimates of the critical displacement velocities of the zone where the nature of the wave motion changed and of the dimensions of the angular zones covered by the waves [9-11]. Hence, the elasticity modulus, the density, and the Poisson coefficient of the ice plate were taken to be $3 \cdot 10^9$ N/m², 870 kg/m³, 0.34, and the density and depth of the liquid were assumed equal to 10^3 kg/m³ and 100 m. The distribution v_{jk} (j = 0, k = 1, 2, 3; j = 1, k = 0, 1, 2) of oscillation frequencies for h = 0.2 m is indicated in Fig. 2, where the dashed and dot–dash lines 1 represent the velocities v_{02} and v_{03}, and lines 2 represent the velocities v_{10} and v_{12}. The dotted line characterizes v_{01}, and the solid line is for v_{11}. The square on the axis σ denotes the value of σ_0.

It follows from Fig. 2 that v_{12} and v_{13} grow with an increase in σ. The velocities v_{01}, v_{02}, and v_{10} then decrease, and v_{11} has a minimum. For motion of a pressure zone with constant intensity along a solid, elastic plate, there are three critical values for v: $v_0 = v_{01} = v_{03}$, $v_1 = v_{11} = v_{12}$, $\sqrt{gH} = v_{10} = v_{02}$. For an absolutely fragile plate (crushed ice) or for an open liquid surface with motion of a periodic pressure zone, the critical velocities will be v_{02} and v_{10}, which decreasse with an increase in σ, and for a pressure zone with constant intensity there is only a single critical velocity $v = \sqrt{gH}$.

The velocities v_{01}, v_{02}, and v_{03} are also critical for displacement of a planar front of periodic pressure zones along an elastic plate (solid ice) [12]. If a planar front of pressure zones with constant intensity moves along an elastic plate, then $v = v_0$ and $v = \sqrt{gH}$ will be critical. In a liquid with an absolutely fragile plate (crushed ice) or with

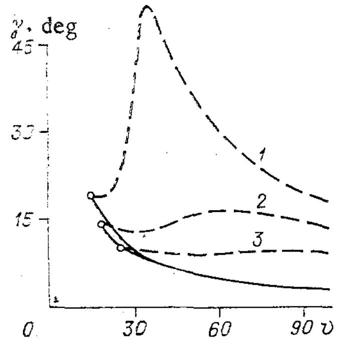

Fig. 4

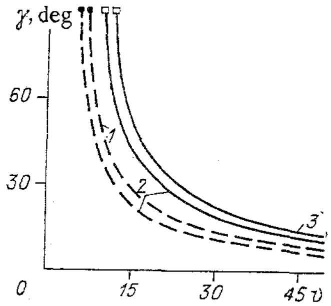

Fig. 5

an open surface, movement of a planar front of periodic pressures results in the critical velocity $v = v_{02}$, while for a planar front of constant pressures $v = \sqrt{gH}$ is critical.

The dimensions of the angular zones covered by the waves as functions of the frequency of the oscillations and of the displacement velocity of the pressure zone are given in Figs. 3-5 for h = 0.2 m.

The velocity distribution γ_k (k = 1, 2, 3) for v is shown in Fig. 3, where the dashed and dot-dash curves denote the values of the frequency $\sigma = 0$ and 0.2 sec^{-1}, and lines 1-3 correspond to γ_1, γ_2, and γ_3. The upper part of the closed solid curve between the triangle and the square represents the maximum values of the angle γ_3, while the lower part pertains to the maximum values of the angle γ_1. These values are attained when $v = v_{02}$. The square and triangle indicate the maximum values of the angles $\gamma_{1,3}$ which pertain to the frequencies $\sigma = 0$ and σ_0. The values indicated by the circles are attained when $v = v_{11}$. The dependences indicated in Fig. 3 show that, with an increase in v, the angles γ_2 and γ_3 decrease while γ_1 increases for frequencies less than σ_0, which is equal to ~0.46 sec^{-1} for the initial parameters given above. The highest frequency corresponds to the smallest angle γ_3 and the largest angle $\gamma_{1,2}$. One should note that for frequencies of $\sigma > \sigma_0$, the angles γ_2 and γ_3 also decrease with an increase in v. An increase in σ will then lead to a decrease not only in γ_3 but in γ_2.

The velocity distributions γ_{11} (dashed lines) and γ_{22} (solid lines) over v are given in Fig. 4, where lines 1-3 pertain to the frequencies 0.01, 0.2, 0.5 sec^{-1}. The values of the angles for $v = v_{12}$ are indicated with circles. These dependences reveal that γ_{22} decreases with an increase in v and σ. The angle γ_{11} also decreases with an increase in the frequency. As a function of v, the angle γ_{11} has extrema whose values and locations are functions of σ. When $\sigma = 0$, the part of curve $\gamma_{11}(v)$ to the right of the maximum coincides with $\gamma_3(v)$, while the part to the left corresponds to the function $\gamma_1(v)$ (see Fig. 3). Angle γ_{22} then coincides with γ_2.

The angles γ_{01} and γ_{02} decrease with an increase in v. This is illustrated in Fig. 5, where $\gamma_{01}(v)$ is indicated by dashed lines and $\gamma_{02}(v)$ is indicated by the solid lines, while lines 1-3 pertain to the frequencies 0.2, 0.4, 0.8 sec^{-1}. The values in the circles are attained when $v = v_{01}$, and those values in the squares are attained when $v = v_{03}$. It also follows from Fig. 5 that an increase in σ leads to a decrease in γ_{01} and an increase in γ_{02}. If $\sigma = 0$, then $\gamma_{01} = \gamma_{02}$.

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VARIATIONAL MODEL OF ORGANIZED VORTICITY IN PLANE FLOW

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In the research of the last decade [mostly experimental (see the review [1]) and numerical (see the bibliography in [2])] a new phenomenon in turbulent flow has been widely studied: that of organized or coherent structures. The characteristic traits of coherent structures that are common in different flows have been formulated. In particular, the primary effect of nonviscous mechanisms on their formation and evolution have been noted. Hence, the analytical models of coherent structures use exact and approximate solutions of the Euler equations for the dynamics of an ideal fluid. However, this approach naturally forces various simplifications, and cannot completely take into account the existing information on coherent structures. For example, in models of shear layers [3-5], chains of coherent structures were considered with a uniform distribution of vorticity inside each of the individual structures. In [3, 5] coherent structures were represented by Kirchhoff and Rankine vortices. In [6-8] the equations for the chains of coherent structures were closed using circular vortices from a single-parameter family [9].

In most of the models of shear layers, the interaction of an individual structure with other coherent structures is taken into account approximately. For example, in [3] the effect of the chain was replaced by a uniform deformation field. In [4, 5] vortices of a given shape were used, and in [6-8] the simplest approximation of point vortices was used.

In the present paper an analytical model of coherent structures in plane flow is constructed by using a variational principle borrowed from information theory. The field of vorticity in the coherent structure is found from the condition that the informational entropy functional be a maximum. In this approach one can use additional constraints to take into account different kinds of information on the basic properties of coherent structures in specific examples, such as dynamical invariants, symmetry properties of the structures, and characteristics of the average flow field.

The variational principle is applied to the problem of a linear chain of coherent structures in an infinite shear layer. The functional equation for the vorticity field in an individual coherent structure is given, in which the nonviscous interactions of the structures are systematically taken into account. It is found that one of the analytical solutions of the equation can be represented in closed form. This is the single-parameter family of Stuart vortices [10]. Using this solution, we construct a model of a chain of coherent structures for a time-dependent shear layer and our model reproduces the general features of its evolution. It is shown that for a certain choice of the family parameter one can obtain, with the help of the Stuart vortices, certain average characteristics of turbulent mixing layers which correspond to experimental data satisfactorily.

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